

Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 1 (Individual)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
			<hr/>			
			Total score			<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. If  $a$  is the number of real solutions of  $\frac{1}{(x+2)(x+3)} = \frac{1}{(x+1)(x+4)}$ , determine the value of  $a$ .

若  $a$  为  $\frac{1}{(x+2)(x+3)} = \frac{1}{(x+1)(x+4)}$  的实数解的数量，求  $a$  的值。

$a =$

2. If  $x$  is a real number and  $b$  is the maximum value of  $-|x-a-9|-|10-x|$ , determine the value of  $b$ .

若  $x$  为实数及  $b$  为  $-|x-a-9|-|10-x|$  的最大值，求  $b$  的值。

$b =$

3. If real numbers  $x$  and  $y$  satisfy  $4x^2 + 4y^2 + 9xy = -119b$ , determine  $c$ , the maximum value of  $xy$ .

若实数  $x$  及  $y$  满足  $4x^2 + 4y^2 + 9xy = -119b$ ，求  $xy$  的最大值  $c$ 。

$c =$

4. If a positive real number  $x$  satisfies  $x^2 + \frac{1}{x^2} = c$ , determine the value of  $d = x^3 + \frac{1}{x^3}$ .

正实数  $x$  满足方程  $x^2 + \frac{1}{x^2} = c$ ，求  $d = x^3 + \frac{1}{x^3}$  的值。

$d =$

Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 2 (Individual)

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			+	Bonus score		<input type="text"/>
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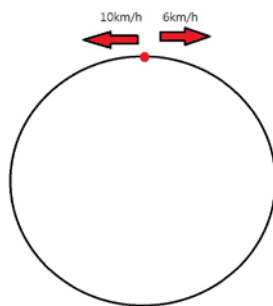
Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
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1. Two students run in opposite directions from a starting point of a 1-km circular track at speeds of 10 km/h and 6 km/h, respectively. They stop running when they meet each other at the starting point again. If  $a$  is the number of times they cross each other after they start and before they stop, determine the value of  $a$ .

两个学生于长 1-km 的圆形跑道的起点开始分别以 10 km/h 及 6 km/h 的速率跑沿相反方向跑步。当他们于起点再相遇时便停止跑步。若  $a$  为他们开始后及停止前相互经过的次数，求  $a$  的值。



$a =$

2. There is a set of red marbles and blue marbles. When  $a$  red marbles are added to the set, the ratio of red marbles to blue marbles is 3:1. When  $a$  blue marbles are added, the ratio of red marbles to blue marbles becomes 2:1. Determine the total number of marbles,  $b$ .

袋中有若干粒红色及蓝色的弹珠。若加入  $a$  粒红色弹珠，红色弹珠与蓝色弹珠的比例为 3:1。若加入  $a$  粒蓝色弹珠，红色弹珠与蓝色弹珠的比例则为 2:1。求弹珠的总数  $b$ 。

$b =$

3. If  $c$  is the smallest difference between 1 000 000 and a square, where the square is a multiple of  $b$ , determine the value of  $c$ .

若  $c$  为 1 000 000 与一个平方数之最小的相差，其中此平方数为  $b$  的倍数，求  $c$  的值。

$c =$

4. The building of a reservoir takes  $d$  technicians, or alternatively  $y$  labours, to complete in a month, where  $d + y = c$ . If  $d$  labours are employed to build the same reservoir, the time taken is 4 times as much as the time taken when  $y$  technicians are employed. Determine the value of  $d$ .

于一个月的时间完成建筑一个水库需要  $d$  个技工或  $y$  个劳工，当中  $d + y = c$ 。若挑选  $d$  个劳工去建筑一个同样的水库，所需要的时间较挑选  $y$  个技工的多 4 倍，求  $d$  的值。

$d =$
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Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 3 (Individual)

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除非特别声明，答案须用数字表达，并化至最简。

1. If  $\{x_0, y_0, z_0\}$  is a solution to the set of simultaneous equations below, determine the value of

$$a = x_0 + y_0 + z_0.$$

$$\begin{cases} 2x - 2y + z = -15 \\ x + 2y + 2z = 18 \\ 2x - y + 2z = -5 \end{cases}$$

若  $\{x_0, y_0, z_0\}$  为以下方程组的解，求  $a = x_0 + y_0 + z_0$  的值。

$$\begin{cases} 2x - 2y + z = -15 \\ x + 2y + 2z = 18 \\ 2x - y + 2z = -5 \end{cases}$$

$a =$

2. Determine the value of  $b = \frac{(\sqrt{6+2\sqrt{a}} + \sqrt{6-2\sqrt{a}})}{2}$ .

求  $b = \frac{(\sqrt{6+2\sqrt{a}} + \sqrt{6-2\sqrt{a}})}{2}$  的值。

$b =$

3. If  $x$  is a positive integer and  $\log_{10} b^x > 3$ , determine  $c$ , the minimum value of  $x$ .

若  $x$  是正整数且  $\log_{10} b^x > 3$ ，求  $x$  的最小值  $c$ 。

$c =$

4. If  $f(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$ , determine the value of  $d = f(c)$ .

若  $f(x) = 2^0 + 2^1 + 2^2 + \cdots + 2^{x-2} + 2^{x-1}$ , 求  $d = f(c)$  的值。

$d =$
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Hong Kong Mathematics Olympiad (2016/2017)  
Final Event 4 (Individual)

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除非特别声明，答案须用数字表达，并化至最简。

1. If  $a$  is a positive integer, determine the greatest value of  $a$  such that  $ax^2 - (a-3)x + (a-2) = 0$  has real roots.

若  $a$  为正整数，求  $a$  的最大值使得  $ax^2 - (a-3)x + (a-2) = 0$  有实根。

$a =$

2. If  $x$  and  $y$  are real numbers with  $1 < y < x$  and  $\log_x y + 3\log_y x = \frac{13}{a}$ , determine the value of

$$b = \frac{x+y^4}{x^2+y^2}.$$

若  $x$  及  $y$  为实数且  $1 < y < x$  及  $\log_x y + 3\log_y x = \frac{13}{a}$ ，求  $b = \frac{x+y^4}{x^2+y^2}$  的值。

$b =$

3. A bag contains  $b+2$  red balls,  $b+3$  white balls and  $b+4$  blue balls. Three balls are randomly drawn from the bag without replacement. Determine the value of the probability,  $c$ , that the 3 balls are of the same colour.

一个袋中有红球  $b+2$  个，白球  $b+3$  个及蓝球  $b+4$  个，从袋中随机抽出 3 个球并不重新放进袋中。求三个抽出的球都是相同颜色的概率  $c$  的值。

$c =$

4. If  $\cos 2\theta = c$ , determine the value of  $d = \sin^4 \theta + \cos^4 \theta$ .

若  $\cos 2\theta = c$ ，求  $d = \sin^4 \theta + \cos^4 \theta$  的值。

$d =$